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Toy Demo

Near-Field Communication with Movable Antennas: An Equilibrium Perspective

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Brief Summary

Optimal antenna placement in near-field LoS wirelss communications for SE maximization [1], [2].

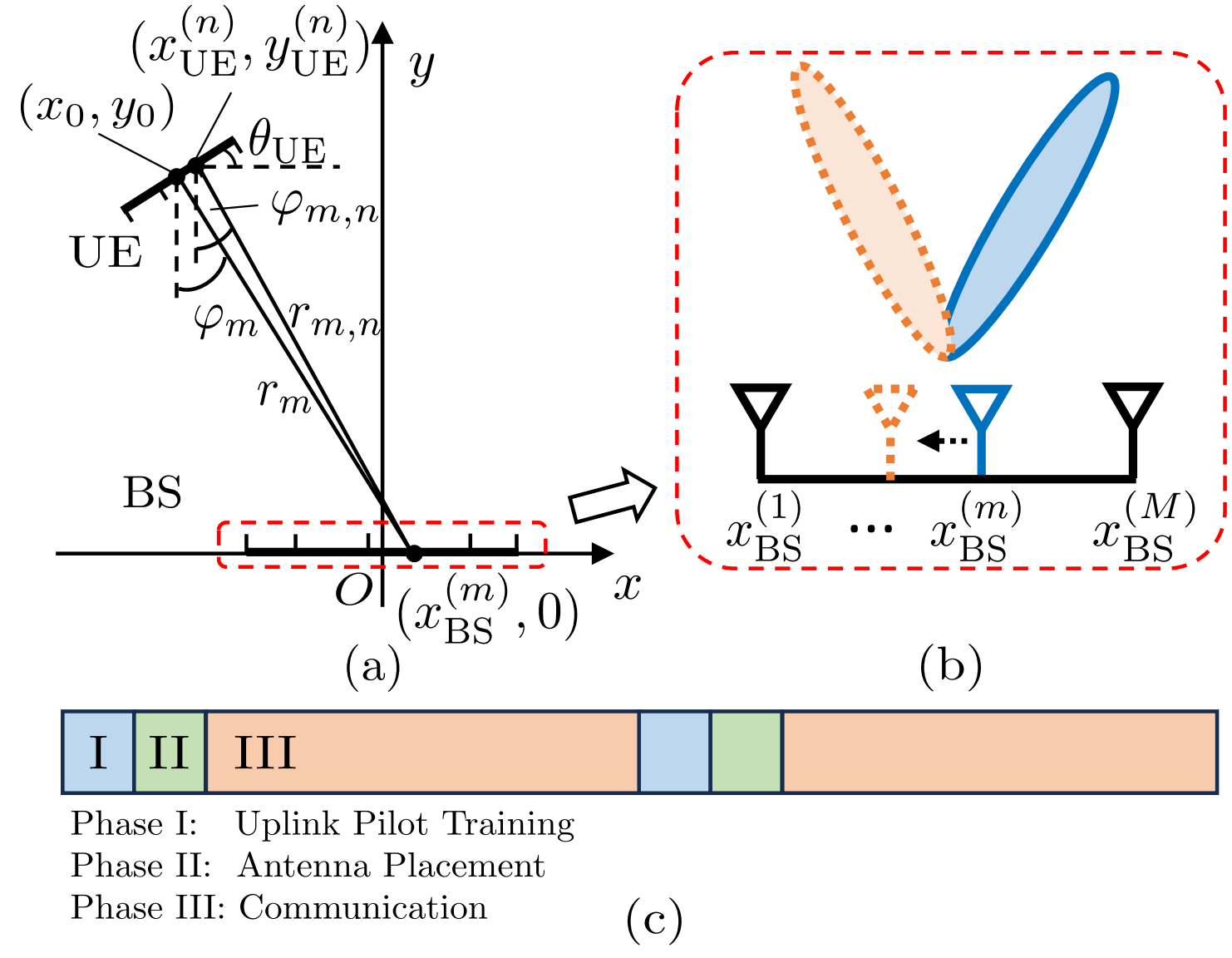
- **Equilibrium** positions with two fixed charges and one external charge.
 - **Greedy** [1]: Approximation via **Leja sequence**
 - **Closed-form** [2]: Roots of polynomial solutions to Stieltjes ODE.

1. Motivations

- **Near field**: Spherical wavefronts over large apertures expose higher spatial DoF that ULA cannot fully utilize.
- **Position Reconfigurable Antennas (MA / FAS / PASS)** sample channel non-uniformly. Utilize DoF by changing the intrinsic channel property.
- Existing placement schemes lacks **Physical Insights** and mostly rely on numerical optimization with **high complexity**.

Goal: A low-complexity, physically interpretable placement rule that unlocks the near-field spatial DoF.

2. System Model



- Aperture $A_T = (M-1)d$, antenna position (ascending order):

$$-\frac{A_T}{2} = x_{BS}^{(1)} < x_{BS}^{(2)} < \dots < x_{BS}^{(M)} = \frac{A_T}{2}, \quad (1)$$

- Near-field LoS channel

$$\mathbf{H}_{\mathbf{x}}[n, m] = \frac{e^{j\pi r_{m,n}}}{r_{m,n}} \quad (2)$$

- SE objective

$$R_{\mathbf{x}} = \log_2 \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H}_{\mathbf{x}} \mathbf{H}_{\mathbf{x}}^H \right). \quad (3)$$

3. Problem Formulation

- For precision, apply **Trigonometric substitution**

$$s_m = \sin(\varphi_m - \theta_{UE}) = \sin \left(\arctan \left(\frac{x_{BS}^{(m)} - x_0}{y_0} \right) - \theta_{UE} \right) = \sin(\varphi_m - \theta_{UE}) \quad (4)$$

- SE maximization problem transformed into a **Weighted Fekete Problem (WFP)** [3]

$$\mathcal{P}_2: \begin{aligned} & \max_{\mathbf{s}} \quad 2 \sum_{1 \leq i < j \leq M} \log_2(s_j - s_i) + \sum_{m=1}^M \log_2(1 - \tilde{s}_m^2) \\ & \text{s.t.} \quad s_{\min} = s_1 < s_2 < \dots < s_M = s_{\max}, \\ & \quad \tilde{s} = \sin(\arcsin s + \theta_{UE}). \end{aligned} \quad (5)$$

Convex and elegant. With lots of target variables, generic interior-point solvers could be slow and give *limited design insight*.

4. Leja Greedy Design [1]

- Place antenna one by one, each at the maximum of the incremental Fekete objective

$$J_D(s) = 2 \sum_{i=1}^{m-1} \log_2 |s_i^* - s| \quad (6)$$

$$J_W(s) = \log_2(1 - \sin^2(\arcsin s + \theta_{UE})).$$

- Derivative-free, low-complexity, near-optimal.

Greedy Antenna Position Design

Input: M , spacing thresholds ϵ , aperture A , UE position (x_0, y_0) and rotation θ_{UE} .

Output: Positions $\{x_{BS}^{(m)}\}_{m=1}^M$ of antennas.

1: Fix endpoints s_1^*, s_M^* at the aperture boundaries

2: **for** $m = 3, \dots, M$ **do**

3: Update the feasible region

$$S_m = \{s \in \mathbb{R} \mid s_1 < s < s_M\} \setminus \bigcup_{i=1}^{m-1} B_{\epsilon},$$

4: Determine the m -th position over S_m

$$s_m = \arg \max_{s \in S_m} (J_D(s) + J_W(s))$$

5: **end for**

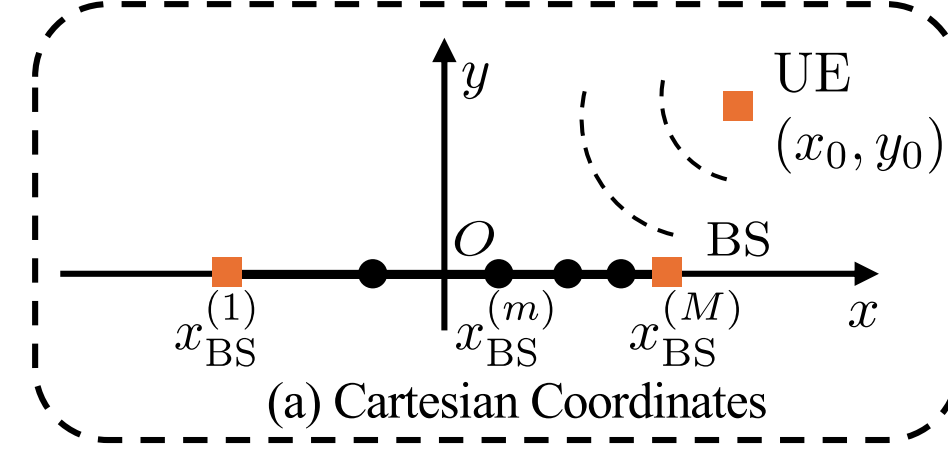
6: Angular positions $\{s_m^*\}_{m=1}^M$ to coordinates $\{x_{BS}^{(m)}\}_{m=1}^M$.

- Execution time [1]

Methods	Computational Complexity ($\mathcal{O}$)	Ave. Exec. Time (ms)
		$M = 16$ $M = 128$
AS	$MG^2(N^3 + M^2N^2)$	47.4 3.38×10^3
Prop. CVX	$\sim M^{4.5}$	6.42×10^3 3.47×10^5
Prop. Greedy	GM^2	2.19 94.1

* Using interior-point method.

5. Electrostatic Equilibrium [2]



- First-order optimality of $\mathcal{P}_2$:

$$\sum_{i=1}^M \frac{2}{s_m - s_i} = \frac{2 \tan \varphi_m}{\sqrt{1 - s_m^2}} \simeq \frac{b}{a - b(s_m - s_0)}, \quad (7)$$

internal force

external force

where $s_0 = \sin(\varphi_0 - \theta_{UE})$, $a = \cos^2 \varphi_0$, and $b = \sin(2\varphi_0) / \cos(\varphi_0 - \theta_{UE})$.

- Apply normalization $t = (s - s_{\min}) / (s_{\max} - s_{\min})$

$$\sum_{i=2}^{M-1} \frac{2}{t_m - t_i} = -\frac{2}{t_m} - \frac{2}{t_m - 1} - \frac{1}{t_m - \tilde{b}}, \quad (8)$$

This is exactly the equilibrium of $M-2$ free charges within $(0, 1)$ repelling each other, with **three fixed charges** at 0, 1, and $\tilde{b}$.

- The optimal placement is the *electrostatic equilibrium* of the charge system.

Physical analogy: M charges on a line, mutually repelling yet pulled by an external charge (the UE) into *equilibrium*.

6. Two-Step EVD-based Solution

- Analytical solution to equilibrium: Polynomial $p(t)$ roots $\{t_m\}_{m=1}^{M-1}$ of Stieltjes ODE [4].

$$\frac{p''(t_m)}{p'(t_m)} = \sum_{i=2}^{M-1} \frac{2}{t_m - t_i} \Rightarrow A(t)p'' + B(t)p' + V(t)p = 0, \quad (9)$$

with $A = t(t-1)(t-\tilde{b})$, $B = 2(t-1)(t-\tilde{b}) + 2t(t-\tilde{b}) + t(t-1)$, and $V = v_1 t + v_0$.

- With only one LoS path, this is the canonical **Heun equation**:

$$p''(t) + \left(\frac{\gamma}{t} + \frac{\delta}{t-1} + \frac{\epsilon}{t-a_H} \right) p'(t) + \frac{\alpha\beta t - \lambda}{t(t-1)(t-a_H)} p(t) = 0, \quad (10)$$

with $\gamma = \delta = 2$, $\epsilon = 1$, $a_H = \tilde{b}$, $\alpha = -(M-2)$, $\beta = M+2$, and $v_1 = \alpha\beta = -(M-2)(M+2)$.

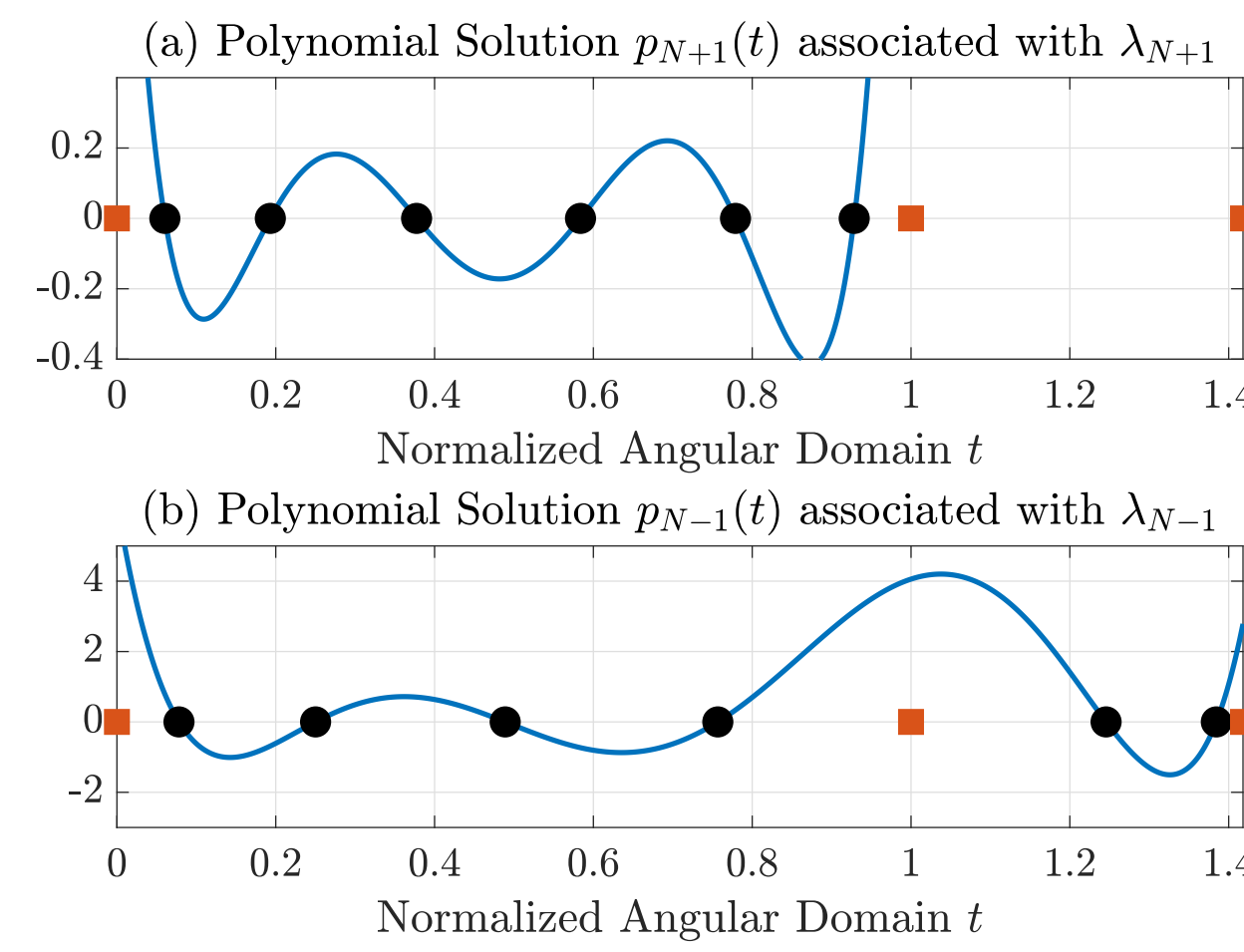
- **EVD Step I**: Build matrix $\mathbf{R}$ and obtain its eigenvector $\mathbf{q}$

$$\mathbf{R} = \begin{bmatrix} B_0 & C_0 & 0 & \dots & 0 \\ A_1 & B_1 & C_1 & \ddots & \vdots \\ 0 & A_2 & B_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & C_{M-2} \\ 0 & \dots & 0 & A_{M-2} & B_{M-2} \end{bmatrix}, \quad \begin{cases} A_n = (n-1+\alpha)(n-1+\beta) \\ B_n = -n((n-1+\gamma)(1+a_H) + a_H\delta + \epsilon), \\ C_n = (n+1)(n+\gamma)a_H \end{cases} \Rightarrow \mathbf{q} \quad (11)$$

- **EVD Step II**: Build matrix $\mathbf{C}$ with $\mathbf{q}$

$$\mathbf{C} = \begin{bmatrix} \mathbf{0}_{1 \times (M-3)} & -\mathbf{q}_{0:M-3} \\ \mathbf{I}_{M-3} & \mathbf{q}_{M-2} \end{bmatrix} \in \mathbb{R}^{(M-2) \times (M-2)} \Rightarrow \boxed{\text{Zeros of } p(t)} \quad (12)$$

- Selecting the desired eigenvector $\mathbf{q}$ is important. Different $\mathbf{q}$ leads to different solutions



- We pick $\mathbf{q}$ according to Sturm-Liouville theory [5]

- $\tilde{b} < 0$: pick the **largest** eigenvalue λ_{M-1} ;
- $\tilde{b} > 1$: pick the **smallest** eigenvalue λ_1 ;
- $0 < \tilde{b} < 1$ (rare): evaluate all $M-1$ eigenvectors and keep the one maximizing $J(\mathbf{s})$.

7. Asymptotic Closed Form

- As $M \rightarrow \infty$ in the near-field region, the internal repulsion dominates

$$\sum_{1 \leq i < j \leq M} \log_2(s_j - s_i) \gg \sum_{m=1}^M \log_2(1 - \tilde{s}_m^2). \quad (13)$$

- The equilibrium condition reduces to the **Jacobi equation**

$$\sum_{i=2}^{M-1} \frac{1}{t_m - t_i} = -\frac{1}{t_m + 1} - \frac{1}{t_m - 1}, \quad (14)$$

$$(1 - t^2) p''(t) - 4t p'(t) + v_0 p(t) = 0, \quad (15)$$

and the exact solution is the Jacobi polynomial

$$p(t) = P_{M-2}^{(1,1)}(t). \quad (16)$$

- It admits a closed form (a.k.a., Chebyshev nodes)

$$t_n = \cos \left(\frac{(2n+1)\pi}{2(M-2)} \right), \quad n = 0, \dots, M-3, \quad \rho(t) = \frac{1}{\pi\sqrt{1-t^2}}. \quad (17)$$

- Transform back to coordinates

- Denormalize

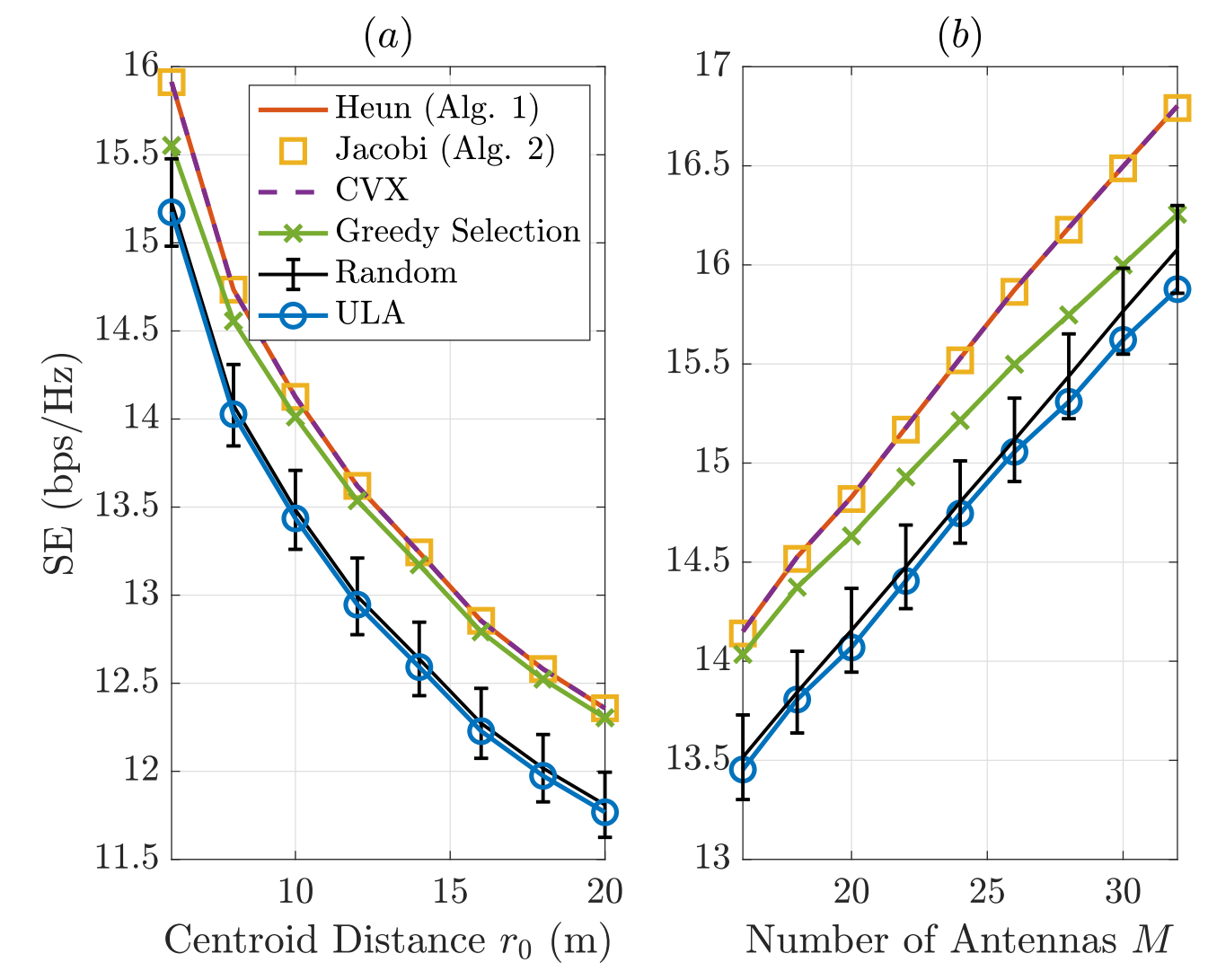
$$s_m = t_m (s_{\max} - s_{\min}) + s_{\min} \quad (18)$$

- Inverse transform

$$x_{BS}^{(m)} = x_0 + y_0 \tan(\theta_{UE} + \arcsin s_m) \quad (19)$$

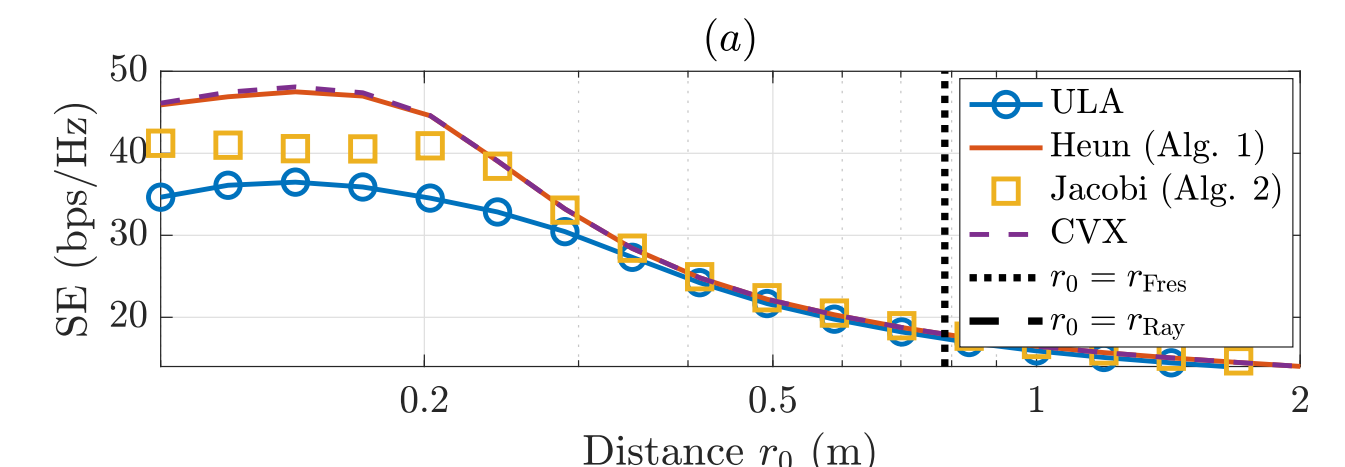
8. Numerical Results

- Setup: $f_c = 10$ GHz, $M = 16$, $N = 8$, spacing $d = 2\lambda$, SNR = 20 dB.
- SE performance with varying distance/number of antennas

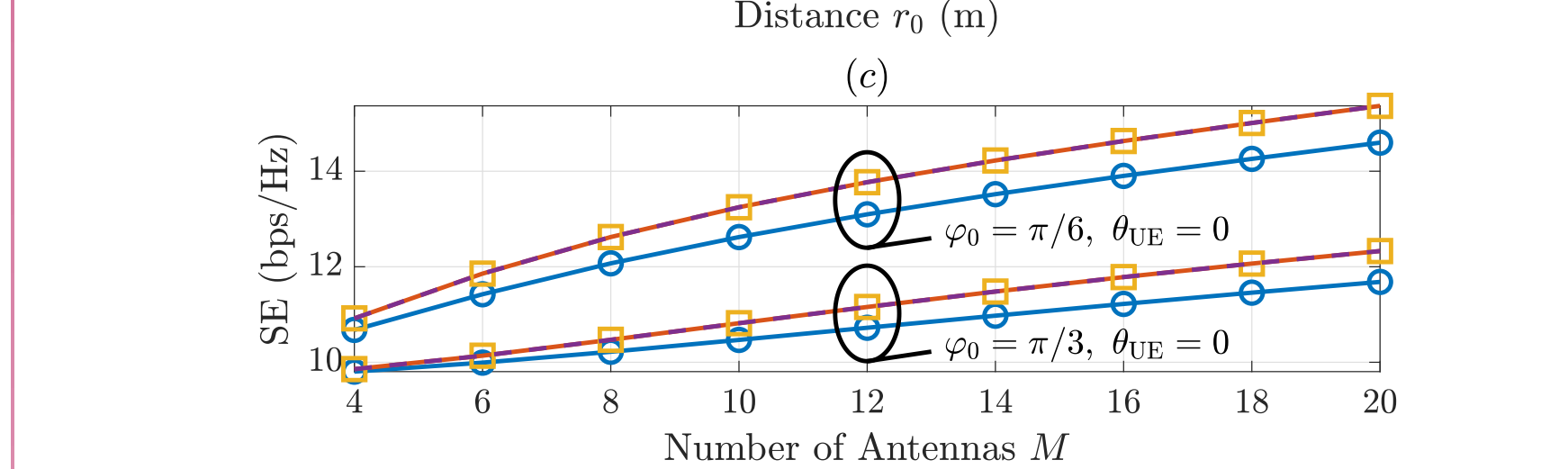


EVD and closed form coincide with **CVX**; ~5~6% gain over ULA.

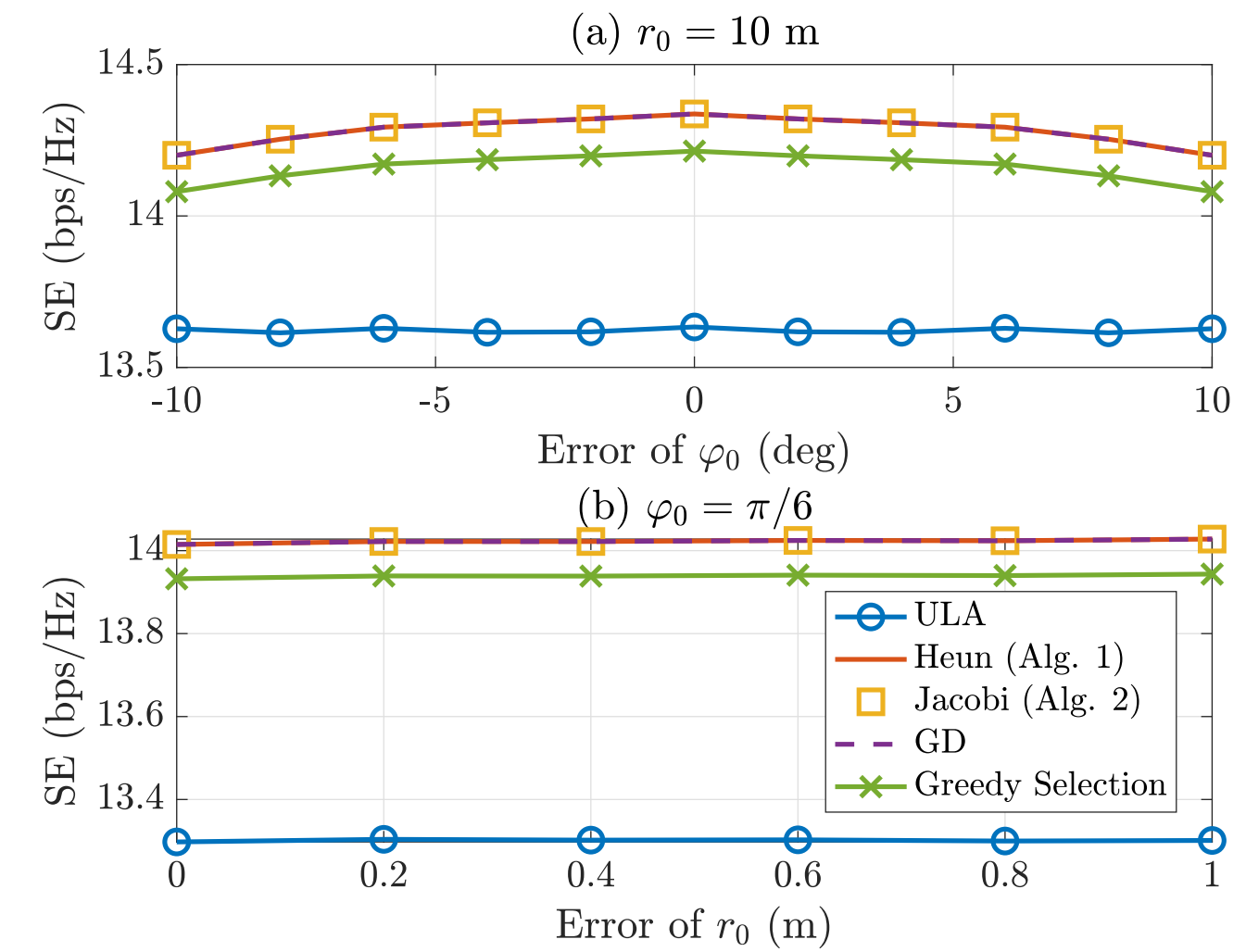
- Asymptotic accuracy



Normalized error $\leq 5\%$ within $r_{\text{Fres}} \leq r_0 \leq r_{\text{Ray}}$; closed form matches EVD from $M = 4$.

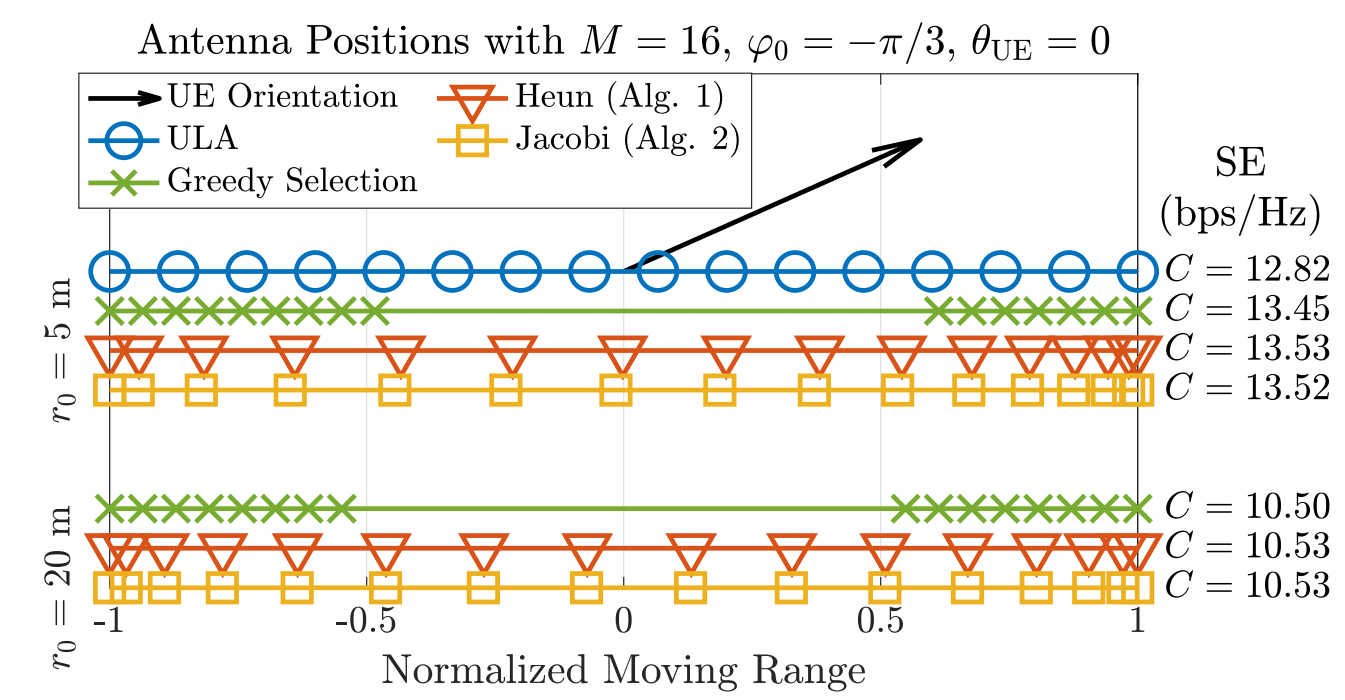


Robustness



10° angle error costs 0.96% SE; Distance mismatch is almost harmless.

- Antenna position patterns



Dense at the edges, sparse at the center; at short range, elements also cluster toward the near UE side.

- Complexity table

Method	Complexity	Avg. time (s)	
		$M = 16$	$M = 64$
EVD (2-step)	$2M^3 + 5M$	7.4×10^{-5}	9.1×10^{-2}
Closed form	—	2.6×10^{-7}	4.1×10^{-7}
CVX	$\sim M^3$	5.2×10^{-2}	2.74
Greedy	$\mathcal{O}(MN^3)$	1.1×10^{-2}	2.2×10^{-1}

EVD: $\sim 30 \times$ faster than CVX at $M = 64$.

Take-Home Messages

- Near-field MA placement is an **electrostatic equilibrium** problem.
- The solution is obtained by performing two-step EVD
- Jacobi approximation shows insignificant performance loss with a closed-form solution.
- The same picture extends to multiuser/ISAC (more external charges) and planar arrays (future works).

References

- [1] S. Liu and X. Yu, "Near-field line-of-sight communication with massive movable antennas," in *IEEE International Conference on Communications (ICC'26)*, Glasgow, Scotland, UK, 2026, pp. 1–6.
- [2] S. Liu, X. Yu, S. Song, and K. B. Letaief, "Near-field communication with movable antennas: An electrostatic equilibrium perspective," *IEEE Trans. Wireless Commun.*, vol. 25, pp. 16 024–16 039, 2026.
- [3] E. B. Saff and V. Totik, *Logarithmic Potentials with External Fields*. Springer Cham, 2024.
- [4] M. Marden, "On Stieltjes polynomials," *Trans. Am. Math. Soc.*, vol. 33, no. 4, pp. 934–944, Oct. 1931, ISSN: 00029947, 10886850.
- [5] M. A. Al-Gwaiz, *Sturm-Liouville Theory and its Applications* (Springer Undergraduate Mathematics Series). Springer London, 2008, ISBN: 978-1-84628-971-2.