

Optimal Placement of Position Reconfigurable Antennas

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Slides & Codes: <https://scliubit.github.io>

October 21, 2026



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Outline

- 1 Background
- 2 Antenna Density (Continuous)
- 3 Antenna Placement (Discrete)
- 4 Conclusions

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Background: Position Reconfigurable Antennas

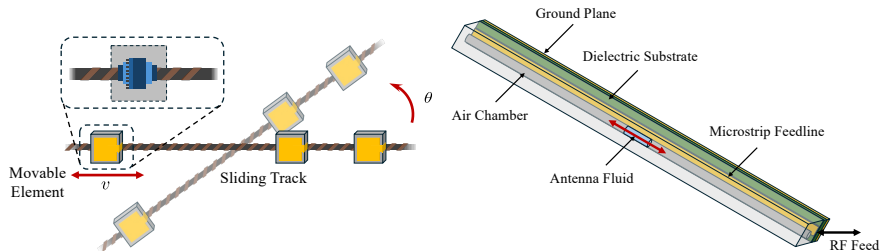


Figure 1: Movable/Pinching antennas [1] (left) and fluid antennas [2] (right).

- **Mechanically Movable:** Slow; Improve channel degree of freedom (DoF).
- **Electronically Fluid:** Fast; Obtain better channel conditions (receive power).

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- [1] L. Zhu, W. Ma, and R. Zhang, "Movable antennas for wireless communication: Opportunities and challenges," *IEEE Wireless Commun.*, vol. 62, no. 6, pp. 114–120, 2024.
- [2] K.-K. Wong, A. Shojaeifard, K.-F. Tong, and Y. Zhang, "Fluid antenna systems," *IEEE Trans. Wireless Commun.*, vol. 20, no. 3, pp. 1950–1962, 2021.

Literature Review

- **Movable Antenna:** Branch and Bound, SCA [3]
- **6DMA:** 3D Position and 3D Rotation, Monte Carlo [4]
- **Fluid Antenna:** Alternating Optimization [5]
- **Pinching Antenna:** MM and Penalty Dual Decomposition [6]
- ...

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- [3] Y. Wu, D. Xu, D. Wing Kwan Ng, W. Gerstacker, and R. Schober, “Globally optimal movable antenna-enabled multiuser communication: Discrete antenna positioning, power consumption, and imperfect csi,” *IEEE Trans. Commun.*, vol. 73, no. 10, pp. 9903–9923, 2025.
- [4] X. Shao, R. Zhang, Q. Jiang, and R. Schober, “6D movable antenna enhanced wireless network via discrete position and rotation optimization,” *IEEE J. Sel. Areas Commun.*, vol. 43, no. 3, pp. 674–687, 2025.
- [5] T. Hao, C. Shi, Q. Wu, B. Xia, et al., “Fluid-antenna enhanced isac: Joint antenna positioning and dual-functional beamforming design under perfect and imperfect csi,” *IEEE Trans. Veh. Technol.*, vol. 74, no. 11, pp. 17 204–17 219, 2025.
- [6] X. Xu, X. Mu, Y. Liu, and A. Nallanathan, “Joint transmit and pinching beamforming for pinching antenna system (pass): Optimization-based or learning-based?” *IEEE Trans. Wireless Commun.*, vol. 25, pp. 11 449–11 464, 2026.

Motivation

- Most previous works deal with antenna placement by numerical optimization.
 - ▶ Limited physical insights
 - ▶ High computational complexity
 - ▶ Near-field Effect ignored

- In this seminar, I will introduce my work that addresses all three challenges

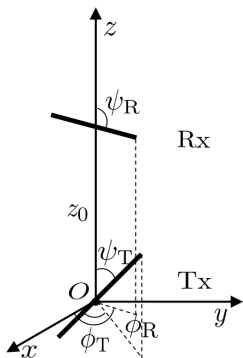
Representative Works

- [7] S. Liu, X. Yu, J. Xu, and R. Zhang, “Near-field communication with massive movable antennas: A functional perspective,” *IEEE Trans. Wireless Commun.*, vol. 25, pp. 14 455–14 470, 2026.
- [8] S. Liu, X. Yu, S. Song, and K. B. Letaief, “Near-field communication with movable antennas: An electrostatic equilibrium perspective,” *IEEE Trans. Wireless Commun.*, vol. 25, pp. 16 024–16 039, 2026.

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System Model



- Point to point wireless system, LoS-dominant channel.
- M movable elements at Tx, N fixed elements at Rx.
- Antennas move freely on a linear array with coordinate

$$\begin{cases} x_T^{(m)} &= \frac{A_T}{2} f(m) \sin \theta_T \cos \phi_T \\ y_T^{(m)} &= \frac{A_T}{2} f(m) \sin \theta_T \sin \phi_T, \quad \forall 1 \leq m \leq M, \\ z_T^{(m)} &= \frac{A_T}{2} f(m) \cos \theta_T \end{cases}$$

where

- ▶ A_T is the overall aperture (move range)
- ▶ Continuous **antenna position function** (APF)

$$p = f(m)$$

denotes the normalized position $-1 \leq p \leq 1$
given index m .

Antenna Density Function

- The number of antenna elements within unit length

$$w(p) = \lim_{\Delta p \rightarrow 0} \frac{f^{-1}(p + \Delta p) - f^{-1}(p)}{\Delta p} = \frac{d}{dp} f^{-1}(p),$$

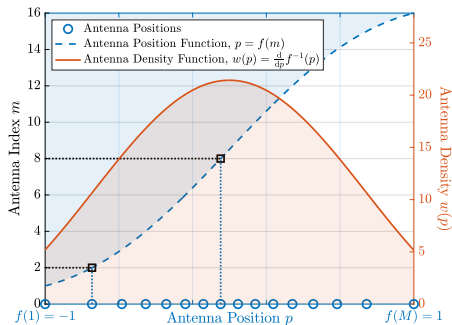


Figure 2: APF $p = f(m)$ (in blue) and ADF $w(p)$ (in orange).

Problem Formulation

- Wireless channel matrix

$$\mathbf{H}_w[m, n] = \frac{e^{-j\kappa r_{m,n}}}{r_{m,n}}$$

is a *functional* of ADF $w(p)$ (or APF $f(m)$).

- Find optimal $w(p)$ to maximize the achievable rate *functional*

$$C_w = \log_2 \det \left(\mathbf{I} + \frac{P_T}{M\sigma_n^2} \mathbf{H}_w \mathbf{H}_w^H \right) = \log_2 \det (\mathbf{T}_N)$$

- **Goal:** To optimize ADF $w(p)$ that maximizes **Toeplitz determinant**.

Problem Formulation

- Each Toeplitz matrix is associated with its unique generating function $s(\theta)$. It generates the column elements via Fourier transform (See [7] for more details).
- (Fresnel approx. and massive M) The generating function of $\mathbf{T}_N$ is

$$s(\theta) = 1 + \frac{2\pi\rho}{\beta z_0^2} w\left(\frac{\theta}{\beta}\right),$$

where $\beta = \frac{\kappa A_T A_R \sin \theta_T \sin \theta_R \cos(\phi_T - \phi_R)}{z_0(N-1)}$ is an antenna geometry-related parameter.

- Goal transfer:** Find $s(\theta)$ that maximizes $\log_2 \det(\mathbf{T}_N)$

[7] S. Liu, X. Yu, J. Xu, and R. Zhang, "Near-field communication with massive movable antennas: A functional perspective," *IEEE Trans. Wireless Commun.*, vol. 25, pp. 14 455–14 470, 2026.

Optimal ADF

- By leveraging the Fisher-Hartwig Theorem [9], [10], the optimal generating function (in angular domain) should be

$$s^*(\theta) \propto |1 - \cos(\theta - \beta)|^\alpha |1 - \cos(\theta + \beta)|^\alpha, \quad -\frac{1}{2} < \alpha < 0$$

- Transform back from angular domain

$$w^*(p) = \left(\frac{\gamma_\alpha}{(1 - p^2)^{-2\alpha}} - \frac{\beta z_0^2}{2\pi\rho} \right) (1 + \tau p)^2 \simeq \frac{\gamma_\alpha}{(1 - p^2)^{-2\alpha}}$$

where γ_α is a normalization factor, τ relates to the angle of arrival.

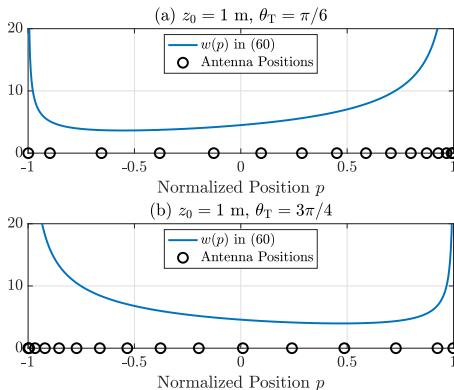
- $\alpha = 0$: ULA; $\alpha = -\frac{1}{4}$: UCA; $\alpha \rightarrow -\frac{1}{2}$: edge-heavy.

[9] M. E. Fisher and R. E. Hartwig, “Toeplitz determinants: Some applications, theorems, and conjectures,” in *Advances in Chemical Physics*. John Wiley & Sons, Ltd, 1969, pp. 333–353.

[10] A. Lenard, “Some remarks on large Toeplitz determinants,” *Pacific Journal of Mathematics*, vol. 42, no. 1, pp. 137–145, 1972.

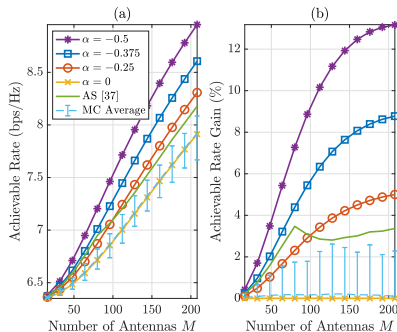
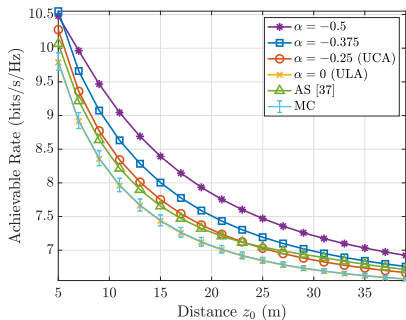
A Glimpse of Antenna Density

- Optimal ADF (and corresponding positions) for different incident directions
- Antennas tend to cluster towards the incident direction



Numerical Results

- Antenna positions have significant effect on achievable rate
- By simply adopting UCA, achievable rate can be improved
- Up to 13% performance gain by closed-form solution

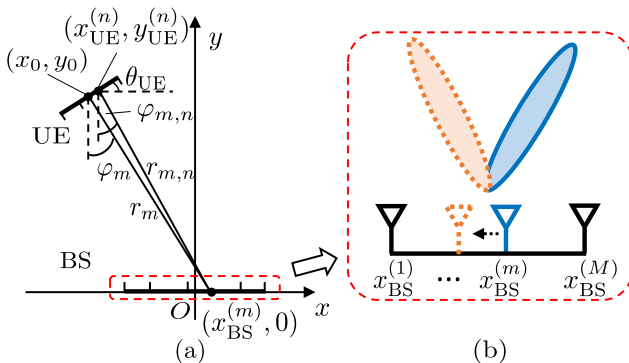


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System Model

- Cartesian coordinate system with trigonometric substitution.
- M Transmit antennas.



Problem Formulation

- Maximize spectral efficiency

$$R_{\mathbf{x}} = \log_2 \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H}_{\mathbf{x}} \mathbf{H}_{\mathbf{x}}^H \right),$$

- Under typical near-field communication settings

$$R_{\mathbf{x}} \simeq \log_2 \det (\mathbf{D}_{\mathbf{T}} \mathbf{D}_{\mathbf{T}}^H) + \log_2 \det \left((\underline{\mathbf{V}}_{\mathbf{T}})^H \underline{\mathbf{V}}_{\mathbf{T}} \right) + \text{const},$$

where

$$\mathbf{D}_{\mathbf{T}} = \text{diag} \left(\left[\frac{e^{j\kappa r_1}}{r_1}, \frac{e^{j\kappa r_2}}{r_2}, \dots, \frac{e^{j\kappa r_M}}{r_M} \right] \right) \quad \Leftarrow \text{Diagonal}$$

$$\mathbf{V}_{\mathbf{T}} = \begin{bmatrix} 1 & \sin(\varphi_1 - \theta_{\text{UE}}) & \sin^2(\varphi_1 - \theta_{\text{UE}}) & \cdots \\ 1 & \sin(\varphi_2 - \theta_{\text{UE}}) & \sin^2(\varphi_2 - \theta_{\text{UE}}) & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ 1 & \sin(\varphi_M - \theta_{\text{UE}}) & \sin^2(\varphi_M - \theta_{\text{UE}}) & \cdots \end{bmatrix} \quad \Leftarrow \text{Vandermonde}$$

Problem Formulation

- Vandermonde determinant maximization problem $\Rightarrow$ **Weighted Fekete problem** [11], [12]

$$\begin{aligned} \max_{\mathbf{s}} \quad & 2 \sum_{1 \leq i < j \leq M} \log_2(s_j - s_i) + \sum_{m=1}^M \log_2(1 - \tilde{s}_m^2) \\ \text{s.t.} \quad & s_{\min} = s_1 < s_2 < \cdots < s_M = s_{\max}, \end{aligned}$$

where $s_m = \sin(\varphi_m - \theta_{\text{UE}})$ and $\tilde{s}_m = \sin \varphi_m = \sin(\arcsin s_m + \theta_{\text{UE}})$

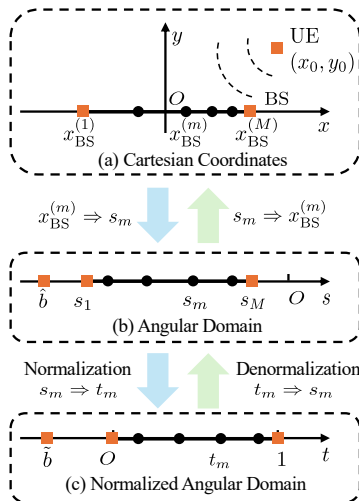
- Proved to be *concave*
- Provide physical insights

[11] E. B. Saff and V. Totik, *Logarithmic Potentials with External Fields* (Grundlehren der mathematischen Wissenschaften), Second. Cham, Switzerland: Springer, 1997, vol. 316, ISBN: 978-3-031-65133-5.

[12] P. Wang, Y. Li, Y. Peng, S. C. Liew, and B. Vucetic, "Non-uniform linear antenna array design and optimization for millimeter-wave communications," *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7343–7356, 2016.

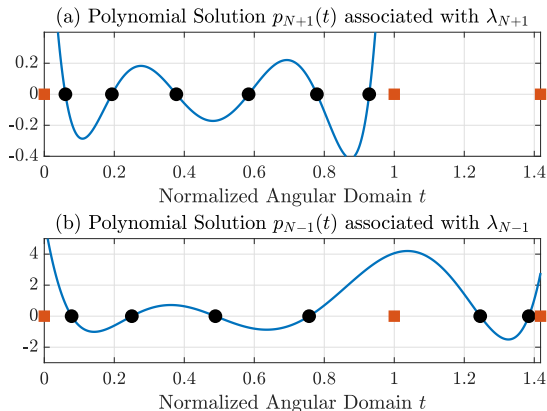
Physical Insight: Electrostatic Equilibrium

- **Free charges:** $M - 2$ antennas, mutual repulsion
- **Fixed charges:** two edge antennas
- **External charge:** determined by UE position
- Optimal placement $\Leftrightarrow$ Equilibrium



Analytical Solution

- The positions of charges are the zeros of the polynomial solution to **Heine-Stieltjes ODE**.
 - ▶ In near-field LoS scenario, it degenerates to Heun equation;
 - ▶ In far-field region, it further degenerates to Jacobi equation



Numerical Solution: Two-Step EVD

- Take Heun equation as example:

$$p''(t) + \left(\frac{\gamma}{t} + \frac{\delta}{t-1} + \frac{\epsilon}{t-a_H} \right) p'(t) + \frac{\alpha\beta t - \lambda}{t(t-1)(t-a_H)} p(t) = 0,$$

- Build matrix $\mathbf{R}$ with Heun parameters, find eigenvector $\mathbf{q}$

$$\mathbf{R} = \begin{bmatrix} B_0 & C_0 & 0 & \cdots & 0 \\ A_1 & B_1 & C_1 & \ddots & \vdots \\ 0 & A_2 & B_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & C_{M-3} \\ 0 & \cdots & 0 & A_{M-2} & B_{M-2} \end{bmatrix} \in \mathbb{R}^{(M-1) \times (M-1)} \Rightarrow \mathbf{q}$$

- Build matrix $\mathbf{C}$ with $\mathbf{q}$

$$\mathbf{C} = \begin{bmatrix} \mathbf{0}_{1 \times (M-3)} & -\frac{\mathbf{q}_{0:M-3}}{\mathbf{q}_{M-2}} \\ \mathbf{I}_{M-3} & \end{bmatrix} \in \mathbb{R}^{(M-2) \times (M-2)} \Rightarrow \boxed{\text{Zeros of } p(t)}$$

- See Appendix and [8] for more details.

[8] S. Liu, X. Yu, S. Song, and K. B. Letaief, "Near-field communication with movable antennas: An electrostatic equilibrium perspective," *IEEE Trans. Wireless Commun.*, vol. 25, pp. 16 024–16 039, 2026.

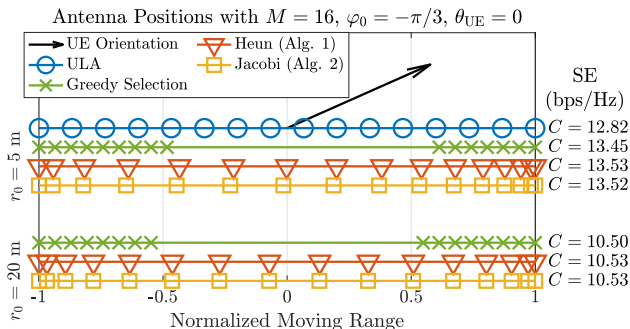
Numerical Solution: Closed-Form Approximation

- When T_x and R_x are not extremely close to each other, the effect of external field can be ignored.
- Degenerate to Jacobi case. Admit close-form solution

$$t = \cos \left(\frac{(2n+1)\pi}{2N} \right), \quad n = 0, 1, \dots, N-1; \quad N = M-2.$$

Numerical Results

- Denser at both edges, shifted toward the UE side (aligns with continuous analysis).
- $r_0 = 5$ m: Heun $\approx$ Jacobi $>$ Greedy $>$ ULA (+5.5% vs. ULA)
- $r_0 = 20$ m: Jacobi (closed form) = Heun (EVD)



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Conclusions

	Part I: Continuous	Part II: Discrete
Variable	ADF $w(p)$	Positions $\{s_m\}$
Structure	Toeplitz	Vandermonde
Theory	Fisher-Hartwig	Electrostatic equilibrium
Solution	Closed-form ADF	Heun / Jacobi zeros

Main observations:

- Optimal Placement: **More antennas near the array edges.**
- Near-field behavior: **Shift towards near-side of UE.**

- [1] L. Zhu, W. Ma, and R. Zhang, “Movable antennas for wireless communication: Opportunities and challenges,” *IEEE Wireless Commun.*, vol. 62, no. 6, pp. 114–120, 2024.
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- [6] X. Xu, X. Mu, Y. Liu, and A. Nallanathan, “Joint transmit and pinching beamforming for pinching antenna system (pass): Optimization-based or learning-based?” *IEEE Trans. Wireless Commun.*, vol. 25, pp. 11 449–11 464, 2026.
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- [8] S. Liu, X. Yu, S. Song, and K. B. Letaief, “Near-field communication with movable antennas: An electrostatic equilibrium perspective,” *IEEE Trans. Wireless Commun.*, vol. 25, pp. 16 024–16 039, 2026.
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- [12] P. Wang, Y. Li, Y. Peng, S. C. Liew, and B. Vucetic, “Non-uniform linear antenna array design and optimization for millimeter-wave communications,” *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7343–7356, 2016.

Thanks for your attention!

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Toeplitz Matrix

Definition 1: Generating Function of Toeplitz Matrix

Let $s(\theta)$ be a real smooth and non-vanishing function defined on unit circle with Fourier coefficients

$$c_k = \frac{1}{2\pi} \int_0^{2\pi} s(\theta) e^{-jk\theta} d\theta, \quad k \in \mathbb{Z}.$$

If a Toeplitz matrix $\mathbf{T}_N$ admits the form as

$$\mathbf{T}_N = \begin{bmatrix} c_0 & c_{-1} & c_{-2} & \cdots & c_{1-N} \\ c_1 & c_0 & c_{-1} & \cdots & c_{2-N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{N-1} & c_{N-2} & c_{N-3} & \cdots & c_0 \end{bmatrix},$$

then $s(\theta)$ is called the **generating function** of $\mathbf{T}_N$.

$$\begin{cases} A_n = (n-1+\alpha)(n-1+\beta) \\ B_n = -n((n-1+\gamma)(1+a_H) + a_H\delta + \epsilon) , \\ C_n = (n+1)(n+\gamma)a_H \end{cases}$$